Quantum effect of electromagnetic potentials on an internal degree of freedom

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We describe a new experimental arrangement in which the so-called quantum effect of potentials (known also as the Aharonov-Bohm effect) is "recorded" on an internal degree of freedom.

INTRODUCTION

The so-called quantum effect of potentials was predicted[1] and later verified experimentally.[2] In trying to understand some of its implications, Aharonov *et al.*[3] introduced a new "language," the language of "modular variables" and "nonlocal interactions." This language permitted for the first time the description of quantum-interference-type experiments in the Heisenberg picture, and also introduced a new viewpoint towards quantum phenomena in general. While much progress (which will be published elsewhere) has been made in developing this approach, we wish to report in this article another example in which the nonlocal aspects of quantum theory manifest themselves by affecting an internal degree of freedom of a charged particle.

The suggested setup. We shall first describe the idea involved here in general terms and afterwards give a specific example. We make use of the general approach, explained in Bohm's book,[4] in which a coherent wave packet is split such that a correlation between the values of an internal degree of freedom and the spatial coordinates is achieved. Then if the separated packets are recombined (as explained in detail elsewhere[5]), they end up in the original coherent wave packet.

To be more specific, let us describe the essential stages of this experiment: A beam of polarized (in the z direction), spin-half[6] charged particles is split and then recombined by using an impulsive interaction of the form $H_I = \mu x \sigma_x$ (regions 1 and 2 in Fig. 1).[5] In mathematical terms, this will be described as follows:

$$|\text{initial}\rangle = |\psi\rangle \otimes \binom{1}{0}, \quad \text{for } t=0 \tag{1}$$

$$|\text{intermediate}\rangle = \frac{1}{\sqrt{2}}\left[|\psi_1\rangle \otimes \binom{1}{1} + |\psi_2\rangle \otimes \binom{1}{-1}\right], \quad \text{for } 0>t>\tau \tag{2}$$

$$|\text{final}\rangle = |\text{initial}\rangle, \quad \text{for } t=\tau \tag{3}$$

where $|\psi\rangle$ is a localized state in configuration space with $\langle x\rangle = \langle p\rangle = 0$, $|\psi_1\rangle$ and $|\psi_2\rangle$ are wave packets with negligible overlap for times $0>t>\tau$ and full overlap for $t=0$ and $t=\tau$, where τ is the total time for the intermediate stage, for which

$$\left.\begin{aligned} \langle x\rangle_1 &\equiv \langle\psi_1|x|\psi_1\rangle = v_0 t \\ \langle p\rangle_1 &\equiv \langle\psi_1|p|\psi_1\rangle = p_0 \\ \langle x\rangle_2 &\equiv \langle\psi_2|x|\psi_2\rangle = -v_0 t \\ \langle p\rangle_2 &\equiv \langle\psi_2|p|\psi_2\rangle = -p_0 \end{aligned}\right\} \quad \text{for } 0<t<\tau/2$$

and

$$\left.\begin{aligned} \langle x\rangle_1 &= v_0\tau - v_0 t \\ \langle p\rangle_1 &= -p_0 \\ \langle x\rangle_2 &= -v_0\tau + v_0 t \\ \langle p\rangle_2 &= p_0 \end{aligned}\right\} \quad \text{for } \tau/2<t<\tau .$$

Note that we have described only the time evolution along the "x axis," the other directions of motion being irrelevant to the argument (see a more detailed mathematical discussion in Bohm's book[4]).

If we now repeat the experiment with a solenoid inserted between the beams as in the Aharonov-Bohm (AB) case,[1] we find that the final direction of the spin depends on the magnetic flux despite the solenoid being in a region inaccessible to the beams. This can be seen from the fact that a relative phase of $(e/\hbar c)\oint \vec{A}\cdot d\vec{s}$ is introduced between the two wave packets during the intermediate stage. If we denote this relative phase by α, we

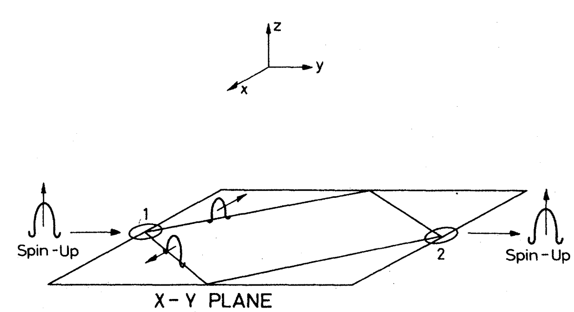

FIG. 1. A beam of polarized, spin-half particles, traveling in the y direction, is split and then recombined. The impulsive interaction $H_I = \mu x \sigma_x$ is applied in regions 1 and 2.

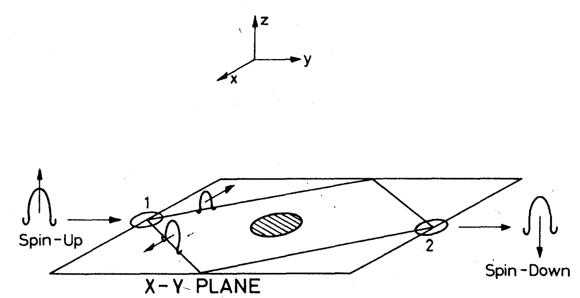

FIG. 2. The solenoid (in the z direction) "introduces" a relative phase of π ($e^{i\pi}=-1$).

can write the expression for the final state as

$$|\text{final}\rangle = |\psi\rangle \otimes \frac{1}{\sqrt{2}}\left[\binom{1}{1} + e^{i\alpha}\binom{1}{-1}\right] \text{ at } t=\tau \tag{4}$$

and if, for example, $\alpha=\pi$, we have "flipped the spin" (Fig. 2) in a deterministic way via a nonlocal interaction. As an example, let us consider the Hamiltonian

$$H=\frac{1}{2m}\left(\vec{p}-\frac{e\vec{A}}{c}\right)^2+\mu\vec{\sigma}\cdot\vec{B},$$

where $\vec{B}$ is a constant magnetic field in the z direction, so that

$$B_z(x,y)=\begin{cases} B_0 & \text{for } -4a \leq x \leq -3a,\ a \leq x \leq 2a,\ -\infty<y<\infty \\ -B_0 & \text{for } -2a\leq x\leq -a,\ 3a\leq x\leq 4a,\ -\infty<y<\infty \\ B_s & \text{for } 0\leq x^2+y^2\leq R^2\ (R\ll a) \\ 0 & \text{elsewhere.}\end{cases}$$

The initial state is a localized wave packet in the x-y plane, around the point (x_0,y_0), with spin up in the x direction and the following conditions:

(1) $a>|x_0|\gg R$, $|y_0|\gg R$, which means that the wave packet is in a free-field region as far as B_s is concerned.

(2) The average velocity is big enough such that any spread of the wave packet is negligible during the experiment.

(3) B_0 is big enough such that the reflected part of the packet (at $x=a$ and $x=-3a$ for $\sigma_3=+1$, and $x=-a$ and $x=3a$ for $\sigma_3=-1$) suffers negligible penetration into the region of constant magnetic field.

With these conditions fulfilled it is straightforward to see (Fig. 3) that the recombination of the separated wave packets is realized at the point $(x=-a, y=-y_0)$,[8] where now the spin state[6] is

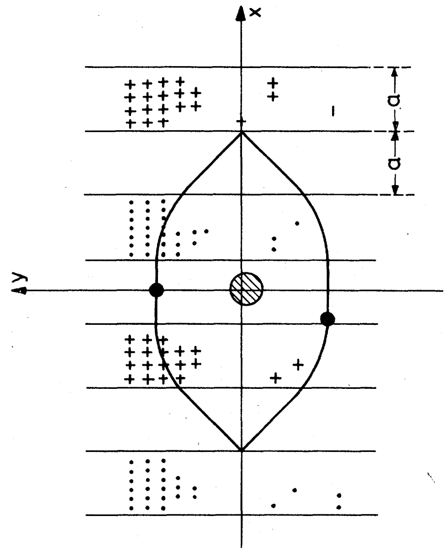

FIG. 3. A beam of polarized (in the x direction), spin-half charged particles traveling in the x direction (in the region $y>0$) is recombined at $x=-a$, $y<0$, where the final spin state is $|\sigma\rangle=(1/\sqrt{2})[\binom{1}{0}+e^{i\alpha}\binom{0}{1}]$, where $\alpha=(e/\hbar c)\oint\vec{A}\cdot d\vec{s}$.

$$|\sigma\rangle=\frac{1}{\sqrt{2}}\left[\binom{1}{0}+e^{i\alpha}\binom{0}{1}\right],$$

in analogy to formula (4) (with the only difference that in this example we have split the beam according to σ_z), where

$$\alpha=\frac{e}{\hbar c}\oint\vec{A}\cdot d\vec{s}=\frac{e\pi}{\hbar c}R^2B_s.$$

CONCLUSION

We have demonstrated the possibility of affecting the direction of the spin of a charged particle using the so-called quantum effect of potentials. This idea can be generalized, in principle, to any internal degree of freedom (e.g., the electric dipole moment of an ionized atom, or even situations involving non-Abelian potentials[7]). We also note that this effect, which is basically an interference-type experiment, can be demonstrated with a single particle.

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*On leave from the University of Tel-Aviv.

[1]Y. Aharonov and D. Bohm, Phys. Rev. 115, 485 (1959). B. S. DeWitt [*ibid.* 128, 2189 (1962)] pointed out that the effect can also be interpreted as a nonlocal effect of the fields.

[2]H. Erlichson, Am. J. Phys. 38, 162 (1970); J. Woodilla and H. Schwarz, *ibid.* 39, 111 (1971).

[3]Y. Aharonov *et al.*, Int. J. Theor. Phys. 2, 213 (1969);

3, 443 (1970).
[4]D. Bohm, *Quantum Theory* (Prentice-Hall, Englewood Cliffs, N.J., 1951), Chap. 22, especially pp. 593–594.
[5]Reference 4, p. 605 and also see R. P. Feynman *et al.*, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, Mass., 1964), Vol. III, Chap. 5.
[6]We choose the representation in which σ_3 is diagonal.
[7]D. Wisnivesky and Y. Aharonov, Ann. Phys. (N.Y.) 45, 479 (1967); T. T. Wu and C. N. Yang, Phys. Rev. D 12, 3943 (1975).
[8]By proper choice of the free parameters of the problem, and the average velocity in the positive x direction.